GREY SYSTEMS THEORY APPLICATIONS TO WIRELESS COMMUNICATIONS

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ABSTRACT

This paper discusses Grey Systems Theory (GST) applications in wireless communications and highlights its potential to cognitive radio. GST consists of information theory concepts and practical algorithms developed to address situations where information is incomplete and affected by random uncertainties. Two GST concepts, Grey Relational Analysis (GRA) and Grey Model (GM) prediction theory are discussed. GRA provides a method to quantify the similarity between a reference data series and set of data while GM is used for modeling time series data and enables prediction of future values with limited data points and unknown probability distributions. These two techniques are surveyed with respect to their applications to wireless communications. Their application to predictive CR and as a similarity measure for case based reasoning cognitive engines is highlighted. A GRA based Automatic Modulation Classification (AMC) algorithm is applied to digital communications signals with preliminary results shown in simulation.

1. INTRODUCTION

Cognitive Radio (CR) is built upon the observation of environmental parameters. Given the stochastic nature of communications and imperfect sensing capabilities, it is reasonable to say that the CR environment consists of *limited data points, and poor information affected by random uncertainties*. In information theory, this is considered a ‘grey’ system in the context of ‘white’ equating to complete knowledge and ‘black’ completely unknown. Techniques that can help ‘whiten’ observed CR meters will be beneficial to improving decision making capabilities.

Grey System Theory (GST) is a family of algorithms developed specifically to address systems with the same characteristics as CR. Developed in China over 20 years ago, GST has seen applications and success across many scientific fields including economics, ecology and transportation [1]. While founded upon information theory philosophy, the practical applications and empirical operation of GST highlight its potential to the mission of CR.

This paper surveys recent GST applications to wireless communications, specifically the Grey Model (GM) prediction theory and Grey Relational Analysis (GRA) algorithms. The potential of these two techniques to CR for proactive/predictive functionality and similarity measures in case based reasoning cognitive engines is highlighted. A new application of GRA to AMC is detailed and demonstrated in simulation.

GRA differs from most similarity measures by incorporating a relational aspect between all the cases in a case database file whereas, in traditional distance measures such as Euclidian Distance, the similarity between each case and the current case is calculated completely independent of the other cases. GRA is closer to models of human memory where all past memories impact recollection.

The literature review indicates that GRA and GM have potential applications in CR. The proposed GRA-based AMC shows promising results in signal classification. It requires no training process and no configuration of proper architecture and parameters as required in other pattern recognition algorithms such as artificial neural network.

The structure of this paper is as follows: Section 2 provides an overview of GST algorithms of GM and GRA. This section surveys the wireless communications application of GST in the literature and presents potential CR uses of GST. Section 3 introduces an application of GRA, GRA-based AMC. The GRA algorithm and AMC process flow is detailed as well as preliminary simulation results are presented. Section 4 summarizes and discusses future research directions.

2. OVERVIEW OF GREY SYSTEMS THEORY

In the early 1980s, Professor Deng Ju-long introduced the concepts of GST. The theories consist of several algorithms designed for studying application spaces characterized by limited data points and modeling information. Two members of the GST overviewed in this section have particular potential towards application in wireless communications and cognitive radio.

2.1 Grey Prediction Theory

The Grey Model (GM) is a member of the Grey System Theory family that provides a tool for modeling discrete series with only a few data points, as little as four in some cases [2]. The model's basis for forecasting lies in identifying an exponential pattern in the data. The GM model is applied to systems with limited data availability and has been implemented in agriculture, earthquake prediction and stock market modeling.

The notation for grey model is GM(n,h), where n is the order of a pseudo differential equation of h variables. The 1st order one variable grey model indicated by GM(1,1) is especially pertinent to forecasting and prediction. The grey differential equation represents discrete (non-continuous) time series. A key assumption of this model is that the discrete data series is exponential or can be manipulated through some level of preprocessing into an exponential pattern. This exponential pattern realized between each element of the series provides the foundations for the grey model. The model attempts to represent the causality between the different elements of the data set leading to a dynamic modeling of a small-sample of discrete data series. Refer to [3] for detailed equations.

2.1.1 Review of Grey Prediction Theory in Wireless Communications

Research of Grey System Theory is growing within the wireless communications domain where it has been used to predict Rayleigh fading mobile communications channels [4]. Rezai, et al. applied grey prediction techniques towards cellular handoff to achieve a significant decrease in handoff delay as well as using a smaller number of handoffs [5]. Compared with traditional hysteresis handoff techniques, the grey prediction methods helped to limit both the mean number of handoffs and hand off delay. Similarly, grey prediction models were used to predict the received signal strength indicator (RSSI) from two base stations [6]. The handoff decision algorithm combined the predicted RSSI, the actual received RSSI value and historical records of past RSSI values to make a decision. Their simulation results indicate minimization of the number of handoffs performed as well as low calculation time.

2.1.2 Grey Prediction for Cognitive Radio

Mitola's original cognitive cycle introduced the use of prediction in his *Plan* phase [7]. Prediction has been explored in the context of forecasting the possible radio configurations [8] and radio resource availability [9]. The GM(1,1) model can estimate the future value or future trend of a time series. The goal of this use of prediction is to input these future values in place of an existing meter in a cognitive radio engine in order to elicit lower latency operations.

2.2 Grey Relational Analysis

The Grey Theory family includes GRA which is designed to provide a quantification of multidimensional distance between an observed data vector and a reference vector. This distance provides a direct correlation to similarity. When compared to a basic similarity function, such as Euclidian Distance, GRA incorporates a relational aspect where the measure of similarity is directly related to all the vectors in the library and not just on the attributes of one row of the library. This relational aspect of the algorithm sets it apart from other methods designed to quantify similarity. GRA can be used in similar fashion to clustering algorithms for determining pattern recognition and matching a signal to known references. Section 3 provides details on the GRA algorithm within an AMC context.

2.2.1 Review of Grey Relational Analysis in Wireless Communications

GRA's ability to identify similarities and assist in decision making has application towards always-best-connected (ABC) heterogeneous communications networks. GRA was used to rank network alternatives to select the best medium based on tradeoffs between quality of service, handoff, and user needs [10]. Other research utilizing GRA for signal similarity matching includes a spectrum identification method for the UM71 signal used in railroad communication [11].

2.2.2 Grey Relational Analysis for Cognitive Radio

Case Based Reasoning (CBR) is proving to be a popular foundation for decision making and learning in cognitive radio [12]. Quantification of similarity lies at the core of CBR. Cognitive radio architectures using CBR match similarity between newly observed situations and past history of radio configurations and associated spectrum conditions. The final selection of new radio parameters are defined through optimization of the most similar past situations from the case library. The ultimate success of the cognitive engine is inherently tied to the strength of the similarity measures.

GRA presents an alternative to Euclidean distance to measure similarity between two vectors. Of particular interest is the relational aspect that GRA brings that incorporates the entire case history when quantifying similarity. Typically, similarity measures quantify case by case where the information in one case has no bearing on another's similarity to the observed situation.

2.3 Limitations of GST

The GM algorithm works with only positive valued data therefore negative data such as dB must first be transposed into positive power ratios or to received signal strength indicator (RSSI). Elbatsh points out that predictive filters such as Kalman filters and Grey Systems theory adapt rapidly to changes in continuous RSSI changes. However, they have difficulty in situations with short disconnects [13].

The primary limitations of GRA include data format, scaling, preprocessing requirements, choice of similarity algorithm, and developing custom weightings. GRA will only work if all the elements in the reference vector are the same type, for example combinations of text and quantified values will not work without transposing the text into some numbered values. Data preprocessing is required to scale all the values between 0 and 1. Refer to [14] for discussion of several scaling methods. This helps GRA mitigate sensitivity to large magnitude differences between the smallest value and largest value in the vector.

Chinese and far eastern researchers dominate the literature in GST but it is slowly expanding its international base. As indicated in [2], poor readability of many papers due to language barriers along with limited theoretical foundations contributes to a reluctance to investigate GST.

Table 1 summarizes the uses, pros, and cons of GST for cognitive radio and wireless communications.

3.0 GREY RELATIONAL ANALYSIS BASED AUTOMATIC MODULATION CLASSIFICATION

3.1 Background

As the demand for spectrally efficient and intelligent communication systems such as software defined radio and cognitive radio grows, the automatic modulation classification (AMC) becomes an important task for recognizing the modulation format of the received signal in an intelligent receiver. AMC algorithms are mainly composed of two steps: signal preprocessing and classification. The classification algorithms are generally classified into either likelihood-based or feature-based [15]. The feature-based algorithms use features of a signal that are distinct for each modulation type and the pattern recognition or the decision theory-based algorithms are then used for classification [15-20].

This GRA AMC method is a feature-based modulation classifier which employs GRA to identify the highest similarity against a library of reference signals. The signal is classified based on this similarity measure. This proposed algorithm is similar to the algorithm for electrocardiogram (ECG) heartbeat recognition using GRA classification whose results showed high accuracy and fast processing time [21]. Lin's method uses the wavelet transform to extract features from the ECG signal and then uses the GRA-based classifier to recognize match against cardiac arrhythmias.

Table 1 Summary of GST for Cognitive Radio

Method	CR Use	Pros	Cons
GM	Proactive decision making	Requires few data points	- Unstable under disconnects - Limited theoretical foundations
GRA	- Similarity Measure CBR - AMC	Incorporates relational aspect	- Requires data preprocessing - Sensitive to magnitude differences

3.2 Method

3.2.1 Feature Extraction

The signal feature used in the proposed GRA-based AMC is an alpha profile (α-profile) because of its distinct pattern for each modulation type.

Majority of communication signals vary periodically with time due to the modulation, the multiplexing, and/or the sampling process. These signals may be modeled as cyclostationary signals. A signal $x(t)$ is defined to be second order cyclostationary if its autocorrelation function $R_x(t, \tau)$ is periodic in time t for each time lag τ. The second order cyclostationarity of the signal can be analyzed in the frequency domain using the spectral correlation density (SCD) and the spectral coherence function (SCF) [22]. The SCD of a signal $x(t)$ is defined as

$$S_{X_T}^{\alpha}(f)_{\Delta t} = \frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2} \frac{1}{T} X_T\left(t, f+\frac{\alpha}{2}\right) X_T^*\left(t, f-\frac{\alpha}{2}\right) dt \qquad (1)$$

where $X_T(t, f)$ is the local Fourier transform of $x(t)$ and α is a cycle frequency or amount of shift in the spectra. The SCF of a signal $x(t)$ is defined as

$$\mathrm{C}_{\mathrm{X}}^{\alpha}(f) = \frac{\mathrm{S}_{\mathrm{X}}^{\alpha}(f)}{\sqrt{S_X^{0^*}(f+\alpha/2) S_X^{0}(f-\alpha/2)}} \qquad (2)$$

The SCDs/SCFs are attractive features since they are highly distinct for different types of modulation with some limitations discussed in [16]. In order to reduce the amount of data used in classification, the highest value of SCF for a given α is used and defined as [16]

$$\alpha\text{-profile} = \max_f [\mathrm{C}_{\mathrm{X}}^{\alpha}(f)] \qquad (3)$$

3.2.2 Enhanced-GRA Classification

This section describes the enhanced GRA classifier which utilizes the GRA algorithm presented in [21] together with normalization and weighting methodologies. Other variations of the GRA algorithm are also available [4].

First, the α-profile of each modulated signal with no noise is generated and converted to a data vector. The library of these α-profiles, called reference vectors, is used to compare with the α-profile of the received noisy signal, called an observed test sequence, for the classification. Figure 1 shows the α-profiles of Binary Phase-shift Keying

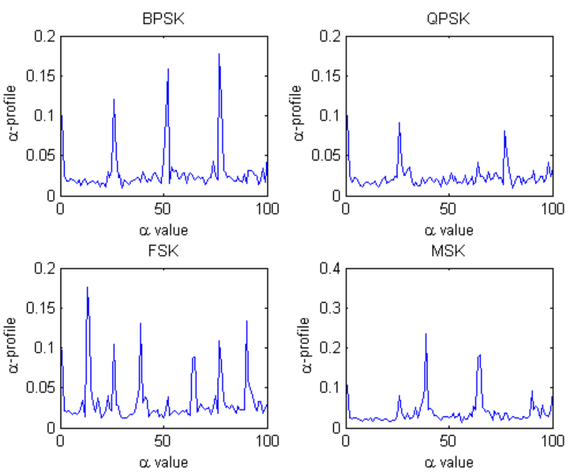

Figure 1 α-profiles for BPSK, QPSK, FSK and MSK

(BSPK), Quadrature Phase-shift Keying (QPSK), Frequency-shift Keying (FSK) and Minimum-shift Keying (MSK) modulations.

The normalization and the weighting methodology are pre-processing techniques used to quantify the importance of each element in the α-profile based on its magnitude. This allows the distinguishable characteristics (peaks) of the α-profile to have more influence over the final decision. Maximum Value (MV) normalization, where each α-profile is normalized by its highest value, is used for this AMC. A weighting function, given in (4) and shown in Figure 2, increases an importance of the peaks by applying a weight that is directly related to the magnitude of the α-profile. The threshold value of 0.25 was selected in the weighting function since we observe that most of the unimportant elements of each α-profile had a magnituide under 0.25.

$$W(x) = \frac{1}{2}\left\{\tanh\left((|x| - 0.25)\cdot 60 - 1\right) + 1\right\} \tag{4}$$

Let the observed test sequence, Φ'_{test}, consist of n sample points $\varphi'_i(0)$, i =1, 2,…n as shown in (5). A library of reference vectors, Φ'_{comp}, consist of K reference vectors where each vector is given as $\Phi'(k) = [\varphi'_1(k), \varphi'_2(k), \ldots, \varphi'_n(k)]$ for $k = 1, 2, \ldots, K$ as indicated in (6).

$$\Phi'_{\text{test}} = [\varphi'_1(0), \varphi'_2(0), \cdots, \varphi'_i(0), \cdots, \varphi'_n(0)] \tag{5}$$

$$\Phi'_{comp} = \begin{bmatrix} \Phi'(1) \\ \Phi'(2) \\ \vdots \\ \Phi'(k) \\ \vdots \\ \Phi'(K) \end{bmatrix} \tag{6}$$

Each vector is normalized as

$$\Phi'(k)_{\text{norm}} = \frac{[\varphi'_1(k), \varphi'_2(k), \cdots, \varphi'_i(k), \cdots, \varphi'_n(k)]}{\max_i \varphi'_i(k)} \tag{7}$$

for $k = 0, 1,\ldots, K$ and $i = 1, 2, \ldots, n$.

For each normalized vector (α-profile), the weighting vector is then calculated and multiplied with the corresponding normalized vector as shown in (8) and (9).

$$\Phi_{\text{test}} = \mathrm{W}(\Phi'(0)_{\text{norm}}) \times |\Phi'(0)_{\text{norm}}| \tag{8}$$

$$\Phi(k)_{comp} = \mathrm{W}(\Phi'(k)_{\text{norm}}) \times |\Phi'(k)_{\text{norm}}|$$

$$= \begin{bmatrix} \varphi_1(1) & \varphi_2(1) & \ldots & \varphi_i(1) & \ldots & \varphi_n(1) \\ \varphi_1(2) & \varphi_2(2) & \ldots & \varphi_i(2) & \ldots & \varphi_n(2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(k) & \varphi_2(k) & \ldots & \varphi_i(k) & \ldots & \varphi_n(k) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(K) & \varphi_2(K) & \ldots & \varphi_i(K) & \ldots & \varphi_n(K) \end{bmatrix} \tag{9}$$

The absolute value of the difference between the test sequence and the k^{th} reference vector is then calculated.

$$\Delta\varphi_i(k) = |\varphi_i(0) - \varphi_i(k)| \tag{10}$$

The next step is to calculate quantification of rankings known as the Grey Relational Grade, as shown in (11)-(13), where the coefficient $\xi \in [0, 5]$. This coefficient is used to weaken the effect of $\Delta\varphi_{max}$ as it grows large. $ED(k)$ is the Euclidian Distance between the test sequence and each reference vector as shown in (14).

$$r(k) = \exp\left[\xi\left(\frac{ED(k)}{\Delta\varphi_{max} - \Delta\varphi_{min}}\right)^2\right] \tag{11}$$

$$\Delta\varphi_{min} = \min_{\forall k}\left[\min_{\forall i} \Delta\varphi_i(k)\right] \tag{12}$$

$$\Delta\varphi_{max} = \max_{\forall k}\left[\max_{\forall i} \Delta\varphi_i(k)\right] \tag{13}$$

$$ED(k) = \sqrt{\sum_{i=1}^{n} (\Delta\varphi_i(k))^2} \tag{14}$$

The Grey Relational Grades, *r(k)*, provide a quantification of similarity between the test vector and the library of reference signals. The higher the relational grade, the higher the similarity and the closer the signal is in term of distance from the reference signal. A relational grade of 1 corresponds to an exact match. Recall, that this similarity metric is a representation of distance away from the reference signal. The power of the GRA is that it provides a multidimensional quantification of distance combined with a relational element encompassing all the signals in the reference library.

3.2.3 Results

Due to limitation in the α-profile where the higher-order QAM and higher-order PSK exhibit the same features as QPSK [22], four modulation types are considered: BPSK, QPSK, FSK, and MSK. The α-profiles shown in Figure 1 are calculated with 100 α-points and a ratio of a carrier frequency to a sampling frequency (Fc/Fs) = 0.2. Note that the synchronization at the receiver is assumed to be perfect, i.e. the carrier frequency and the symbol rate of the received signal can be correctly obtained.

The proposed enhanced GRA classifier outputs the modulation type after it compares the normalized and/or weighted α-profile of the noisy received signal with the normalized and/or weighted α-profiles of four noiseless modulated signals (reference vectors). The Monte Carlo simulation results for the proposed GRA-based AMC with weighting function are shown in Table 2 for different SNRs varying between -5 dB and 15dB. Figures 3 and 4 show the Probability of Correct detection (Pc) with various SNR (dB) with and without weighting function for all modulation formats.

Table 2 GRA-based AMC Confusion Matrix

Tx / Rx	BPSK	QPSK	FSK	MSK
BPSK	1229	0	0	0
QPSK	33	1200	0	22
FSK	0	0	1178	118
MSK	0	0	0	1220

3.2.4 Discussion / Limitations

We can see in Figures 3 and 4 that the results are promising especially for SNR above 0 dB. The weighting improves the results for Pc of BPSK, FSK, and MSK at lower SNR. However, Pc of QPSK was negatively affected. It is likely that when SNR is low, noise-induced peaks in the QPSK α-profile of the received signal will make it appear to be that of a different modulation type.

As SNR decreases, the peaks of the α-profile of the received signal may decrease or change. Distinguishable peaks may still be noticeable, but the Euclidian distance between the observed profile and each reference profile will be large, and therefore a small difference in the noise can easily force the GRA to classify the signal incorrectly. The first step in pre-processing should therefore be normalization in order to have both the reference profile and the observed profile on the same scale. This allows a focus on the relative shape of the profile, and not only the

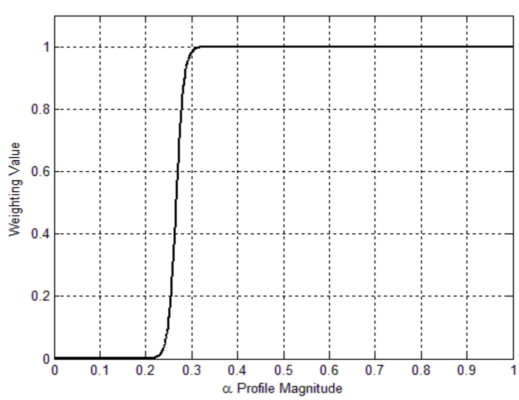

Figure 2 Weighting Function

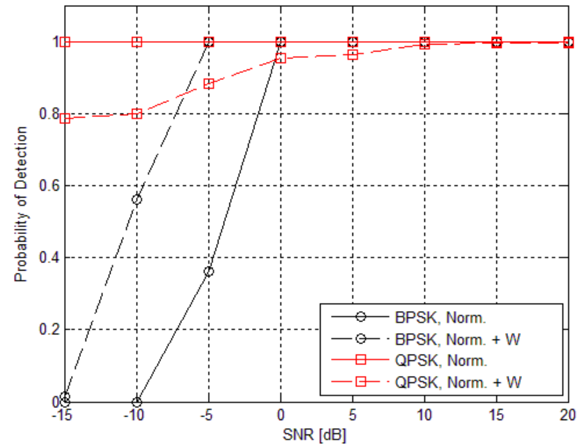

Figure 3 Probability of Correct Detection for BPSK and QPSK

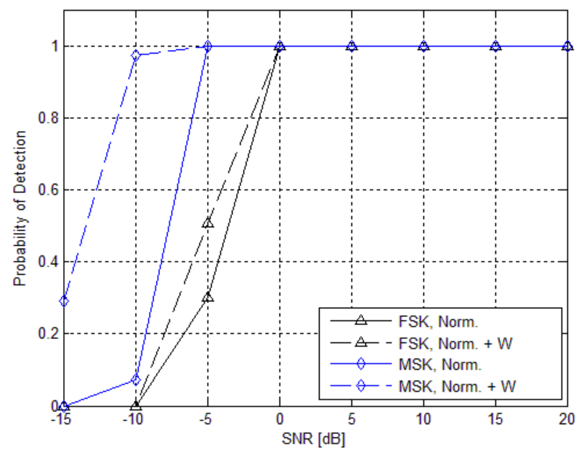

Figure 4 Probability of Correct Detection for FSK and MSK

magnitude. A weighting function increases an importance of the peaks by applying a weight that eliminates unimportant elements and maintains peaks of the α-profile.

The simulation results in Table 2 are comparable to those obtained in [16] when a signal carrier and bandwidth can be correctly obtained. The algorithm of Fehske, et al. uses the α-profile of signals with the artificial neural network for classification. The proposed GRA-based AMC requires no training and no configuration of architecture and parameters as required in the neural network. However, additional preprocessing algorithms must be incorporated for cases when there is no prior knowledge of the signal or when there are offsets in carrier frequency and symbol rate which make the α-profile different from ones in the library of references especially peak locations.

4. SUMMARY

This paper has presented a brief overview of Grey Systems Theory focused on application to wireless communications and cognitive radio. The GM(1,1) model of time series prediction has potential towards aiding in proactive decision making while CBR based engines can utilize GRA for similarity measures. A specific application of GRA towards AMC was also presented.

The proposed AMC has shown promising classification performance in using the enhanced GRA algorithm to classify four modulation types based on the α-profiles. There are clear advantages to normalization and weighting. Future efforts will include investigating other normalization methods that can improve Pc at low SNR and investigating the effect of substituting different similarity metrics in place of Euclidian Distance in the GRA algorithm itself, creating a “Modified GRA”.

The numbers of modulation type in this paper are restricted due to the limitation of the α-profiles. Other signal features and their combinations will be investigated in order to classify more modulation types. The combination of several other signal features can be used with few additional computations in the GRA algorithm. Future efforts will also include integrating other pre-processing algorithms and/or signal features for a case when there are offsets in carrier frequency and symbol rate.

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